Project Report

Te421 – Telecommunications: Principles and Link Budgets

Study of Passband Transmission Systems

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Promotion 2027 – Aero 4
May 2026
Academic Year 2025–2026

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Introduction

This report presents the study and simulation of digital passband transmission systems over an Additive White Gaussian Noise (AWGN) channel. All simulations were carried out in MATLAB.

The project is divided into four parts. Part I implements a complete QPSK (Quadrature Phase Shift Keying) passband communication chain. Part II derives and validates the equivalent lowpass formulation of the same chain. Part III extends the study to three additional modulation schemes – 4-ASK, 8-PSK and 16-QAM – and compares their Bit Error Rate (BER) and spectral efficiency against QPSK. Finally, Part IV verifies the equivalence between the passband and lowpass representations for each of these modulations.

The key parameters used throughout the project are summarised in Table .1.

Table .1: Global simulation parameters

Parameter	**Value**	**Description**
F_s	10 000 Hz	Sampling frequency (Parts I–II)
R_b	2 000 bps	Bit rate (Parts I–II)
M	4	QPSK modulation order
$k = \log_2 M$	2 bits/symbol	Bits encoded per symbol
$R_s = R_b/k$	1 000 symb/s	Symbol rate
$T_s = 1/R_s$	1 ms	Symbol duration
$N_s = F_sT_s$	10	Oversampling factor
α	0.35	Root Raised Cosine roll-off factor
f_c	2 000 Hz	Carrier frequency (Part I)
F_s	12 000 Hz	Sampling frequency (Parts III–IV)
R_b	6 000 bps	Bit rate (Parts III–IV)

Part I – QPSK Passband Chain

Question 1 – Pulse shaping and transmitted signal

1.1 Gray Mapping

The incoming bit stream is grouped into pairs and mapped to QPSK symbols using Gray coding. The constellation is normalised so that each symbol carries unit average power ($\mathbb{E}[|s|^2] = 1$), which is essential for the noise formula to be consistent with the E_b/N_0 definition:

$$00 \to \frac{1+j}{\sqrt{2}}, \quad 01 \to \frac{-1+j}{\sqrt{2}}, \quad 11 \to \frac{-1-j}{\sqrt{2}}, \quad 10 \to \frac{+1-j}{\sqrt{2}} \tag{I.1}$$

Gray coding ensures that adjacent symbols differ by only one bit, which minimises the BER in the presence of noise.

```
Fs = 10000;  Rb = 2000;  M = 4;
k = log2(M);  Rs = Rb/k;  Ts = 1/Rs;
Ns = Fs*Ts;  fc = 2000;  alpha = 0.35;  span = 10;

bits = randi([0 1], 1, 2000);
const_map = (1/sqrt(2)) * [1+1j, -1+1j, -1-1j, 1-1j];

bits_mat  = reshape(bits, 2, []);
idx       = bi2de(bits_mat', 'left-msb');
symbols   = const_map(idx + 1);

h_rrc = rcosdesign(alpha, span, Ns, 'sqrt');
xe_up = upsample(symbols, Ns);
xe    = filter(h_rrc, 1, xe_up);    % complex envelope xe(t)
xI    = real(xe);   xQ = imag(xe);
```

Listing I.1: Gray mapping and RRC pulse shaping

1.2 Root Raised Cosine Filter and Modulation

The pulse shaping filter is a Root Raised Cosine (RRC) with roll-off $\alpha = 0.35$ and span of 10 symbol periods. At the receiver, a matched RRC filter is applied, so that both filters together form a Raised Cosine (RC) filter satisfying the Nyquist inter-symbol interference (ISI)-free criterion. The one-sided occupied bandwidth is:

$$B = \frac{R_s}{2}(1+\alpha) = \frac{1000}{2} \times 1.35 = 675 \text{ Hz} \tag{I.2}$$

The baseband complex envelope $x_e(t) = x_I(t) + jx_Q(t)$ is then transposed to the carrier frequency f_c to produce the real passband signal:

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) = \mathrm{Re}\{x_e(t) \cdot e^{j2\pi f_c t}\} \tag{I.3}$$

The I and Q channels are orthogonal (their inner product over one period is zero), which allows two independent data streams to be transmitted simultaneously on the same frequency band, doubling the spectral efficiency compared to BPSK.

```
t = (0 : length(xI) - 1) / Fs;
x = xI .* cos(2*pi*fc*t) - xQ .* sin(2*pi*fc*t);
```

Listing I.2: Carrier modulation

Result

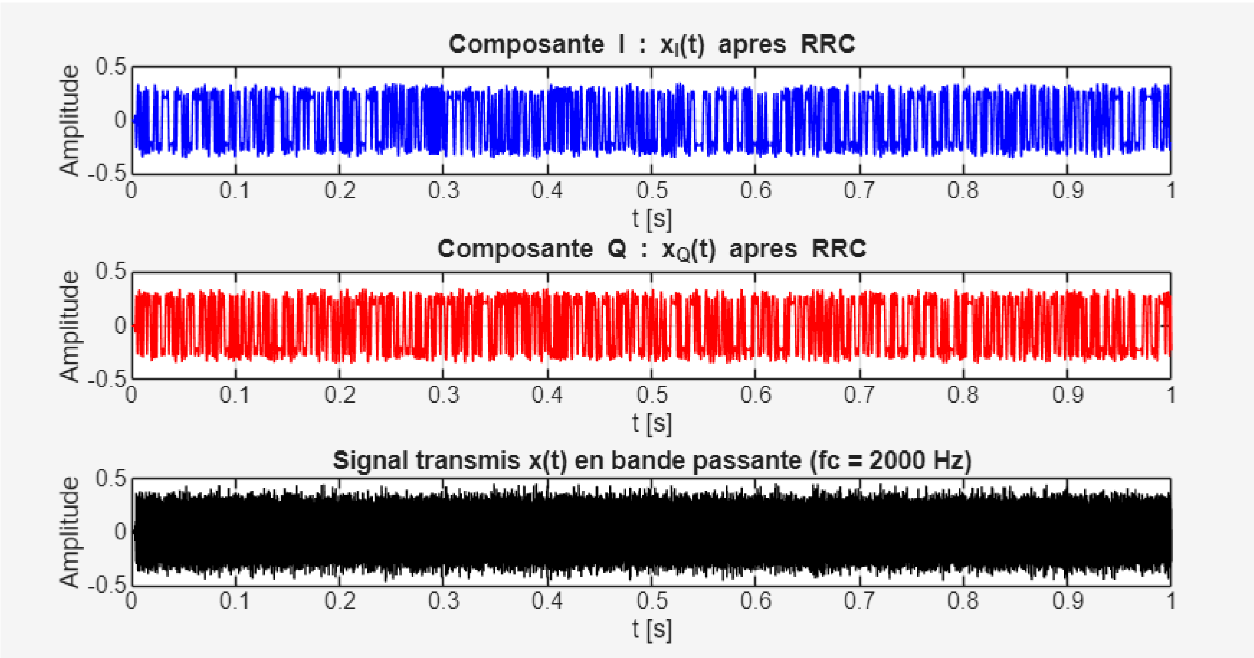

Figure I.1: Pulse-shaped I/Q components and transmitted passband signal $x(t)$ – Part I, Q1.

Question 2 – Power Spectral Density of $x(t)$

The Power Spectral Density (PSD) of $x(t)$ is estimated using Welch's method. Since $x(t)$ is a real passband signal centred on f_c, its spectrum should exhibit two symmetric lobes around $\pm f_c = \pm 2000$ Hz, each with a Raised Cosine shape of half-bandwidth $B = 675$ Hz.

```
[Pxx, f] = pwelch(x, 256, 128, 1024, Fs, 'centered');
plot(f, 10*log10(Pxx));
xlabel('Frequency [Hz]'); ylabel('PSD [dB/Hz]');
title('Power Spectral Density of x(t)'); grid on;
```

Listing I.3: PSD estimation of x(t) using Welch's method

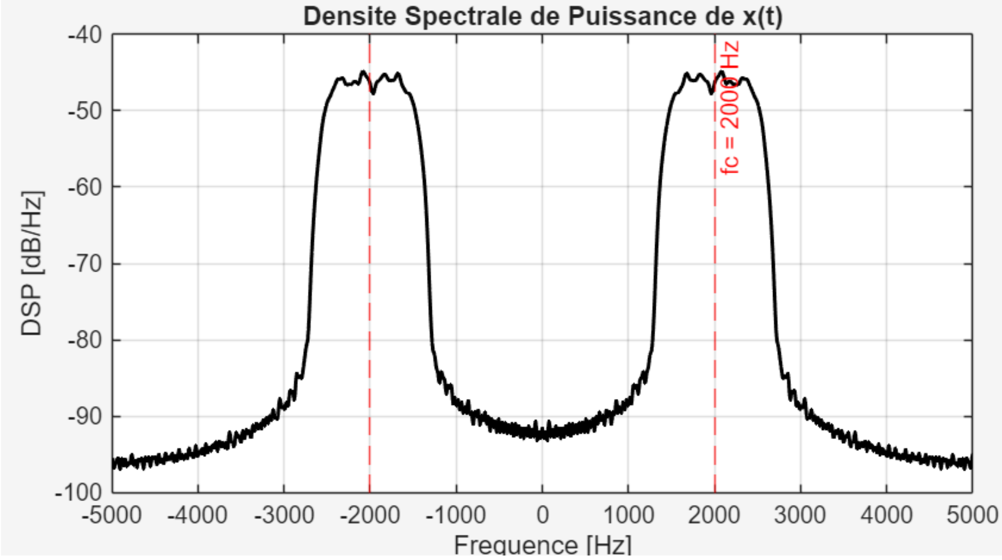

Figure I.2: Power Spectral Density of the transmitted passband signal $x(t)$ – Part I, Q2.

Question 3 – Noise-free chain validation

Before adding noise, the complete chain is run without any perturbation to verify correct implementation. The BER must be exactly zero, as no errors can occur in the absence of noise. A non-zero BER at this stage would indicate a bug in the filter delay compensation, the symbol mapping, or the decision thresholds.

The critical parameter is the RRC filter delay n_0, which shifts the peak of the matched filter response:

$$n_0 = \frac{\text{span} \times N_s}{2} = \frac{10 \times 10}{2} = 50 \text{ samples} \tag{I.4}$$

Optimal sampling instants are then $n_0 + m \cdot N_s$ for $m = 1, 2, \ldots$

```
n0    = span * Ns / 2;         % filter delay = 50 samples
debut = n0 + Ns + 1;           % first valid sampling instant

yI_nb = x .* (2*cos(2*pi*fc*t));
yQ_nb = x .* (-2*sin(2*pi*fc*t));
rI_nb = filter(h_rrc, 1, yI_nb);
rQ_nb = filter(h_rrc, 1, yQ_nb);

instants   = debut : Ns : (length(rI_nb) - n0);
rI_s = rI_nb(instants);  rQ_s = rQ_nb(instants);

sI = sign(rI_s);  sQ = sign(rQ_s);
bits_I    = (1 - sI) / 2;
bits_Q    = (1 - sQ) / 2;
bits_dec = reshape([bits_I; bits_Q], 1, []);
BER_noisefree = sum(bits_dec ~= bits(1:length(bits_dec))) / length(
   bits_dec);
% Expected result: BER_noisefree = 0
```

Listing I.4: Coherent demodulation and decision (noise-free validation)

Validation result: The BER without noise is **0**, confirming the correct implementation of the mapping, filter delay compensation, demodulation, and decision logic.

Question 4 – BER vs E_b/N_0: Simulation and Theory

4.1 AWGN Channel Model

Additive White Gaussian Noise is added to the passband signal $x(t)$. The noise standard deviation σ_n is derived from the desired E_b/N_0 ratio using the relation given in the project statement (Equation 1):

$$\sigma_n^2 = \frac{P_x \cdot N_s}{2\log_2(M) \cdot \frac{E_b}{N_0}} \tag{I.5}$$

where $P_x = \text{mean}(|x|^2)$ is the signal power, $N_s = 10$ is the oversampling factor, and $\log_2(M) = 2$ for QPSK. The factor of 2 in the denominator accounts for the real nature of the passband noise.

4.2 Theoretical BER for QPSK

The theoretical BER for Gray-coded QPSK over an AWGN channel is:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{I.6}$$

This result follows from the fact that QPSK can be decomposed into two independent BPSK streams on the I and Q channels, each with energy E_b.

```
EbN0_dB   = 0:1:8;
EbN0_lin  = 10.^(EbN0_dB/10);
Px        = mean(abs(x).^2);
BER_theo  = 0.5 * erfc(sqrt(EbN0_lin));    % theoretical QPSK BER

for i = 1:length(EbN0_dB)
    sigma_n = sqrt((Px*Ns) / (2*k*EbN0_lin(i)));
    y = x + sigma_n * randn(1, length(x));

    yI = y .* (2*cos(2*pi*fc*t));
    yQ = y .* (-2*sin(2*pi*fc*t));
    rI = filter(h_rrc, 1, yI);  rQ = filter(h_rrc, 1, yQ);

    instants = debut : Ns : (length(rI) - n0);
    sI = sign(rI(instants));  sQ = sign(rQ(instants));
    bits_dec = reshape([(1-sI)/2; (1-sQ)/2], 1, []);
    BER_sim(i) = sum(bits_dec ~= bits(1:length(bits_dec))) / length(
       bits_dec);
end
```

Listing I.5: BER simulation loop and theoretical reference

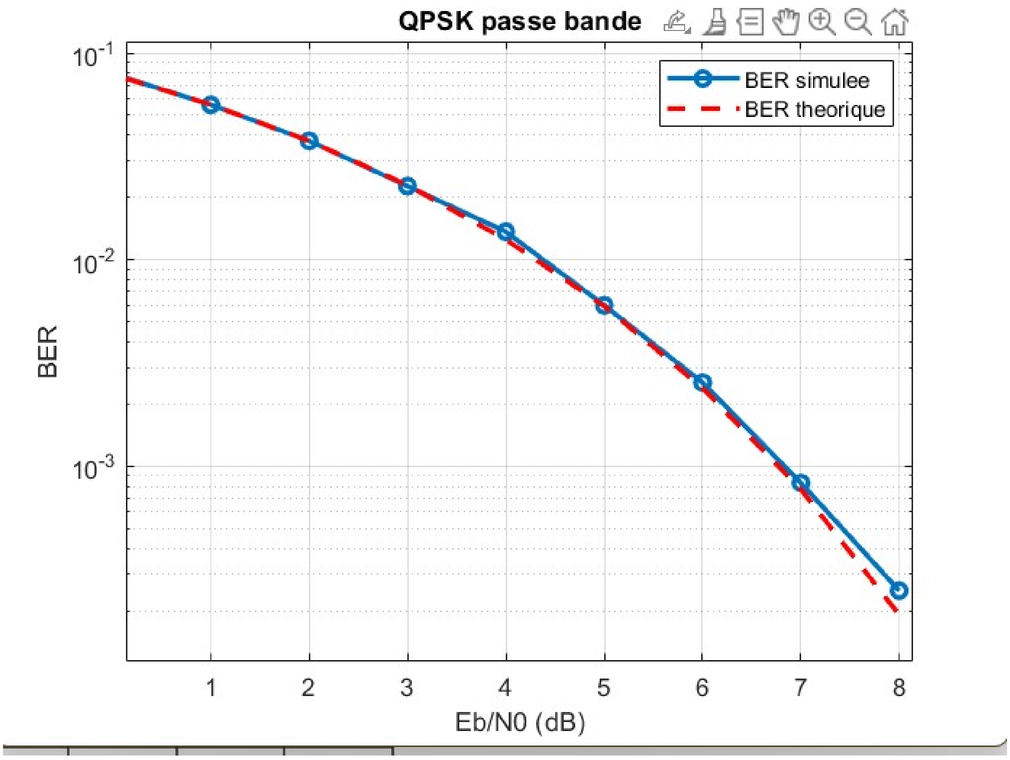

Figure I.3: BER vs. E_b/N_0 for the QPSK passband chain – Part I, Q4.

The simulated curve is expected to closely follow the theoretical expression (I.6), with minor statistical fluctuations due to the finite number of bits simulated ($N_{\text{bits}} = 2000$). At $E_b/N_0 = 8$ dB, the theoretical BER is approximately 1.9×10^{-4}.

Part II – Equivalent Lowpass QPSK Chain

Motivation

The passband chain requires $F_s > 2(f_c + B)$. For a carrier at $f_c = 2$ GHz (as in 4G LTE), the sampling frequency would need to exceed 4 GHz, making simulation computationally intractable. The solution is to work with the complex envelope $x_e(t)$ directly, which is a lowpass signal independent of f_c:

$$x(t) = \mathrm{Re}\{x_e(t) \cdot e^{j2\pi f_c t}\} \tag{II.1}$$

The BER performance of the equivalent lowpass chain is *rigorously identical* to that of the passband chain, as demonstrated by the simulations below.

Question 1 – I/Q signals after pulse shaping

The complex envelope $x_e(t)$ is already computed in Part I. Its real and imaginary parts represent the I and Q baseband channels respectively.

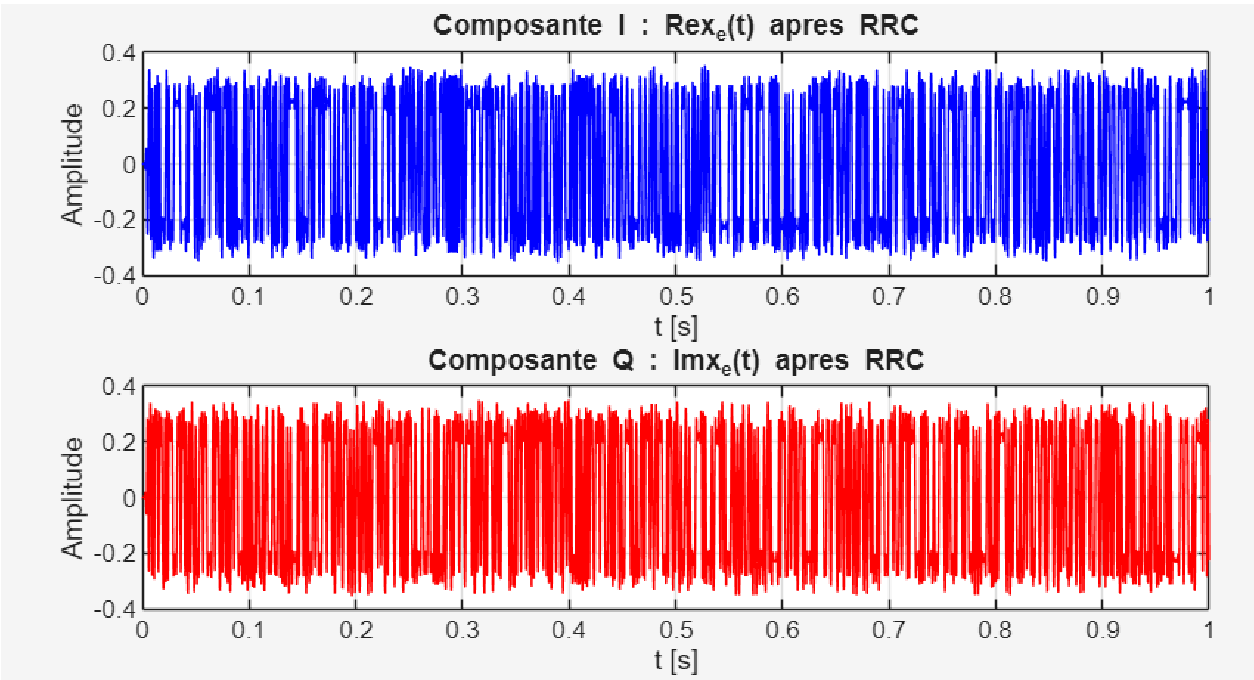

Figure II.1: In-phase and quadrature components of the complex envelope $x_e(t)$ – Part II, Q1.

Question 2 – Power Spectral Density of $x_e(t)$

Unlike the passband PSD centred around $\pm f_c$, the PSD of $x_e(t)$ is centred at 0 Hz and exhibits a single-lobe Raised Cosine shape of half-bandwidth $B = 675$ Hz.

```
[Pxe_spec, f_xe] = pwelch(xe, 256, 128, 1024, Fs, 'centered');
plot(f_xe, 10*log10(Pxe_spec));
xlabel('Frequency [Hz]'); ylabel('PSD [dB/Hz]');
title('PSD of complex envelope x_e(t) -- centred at 0 Hz');
```

Listing II.1: PSD of the complex envelope

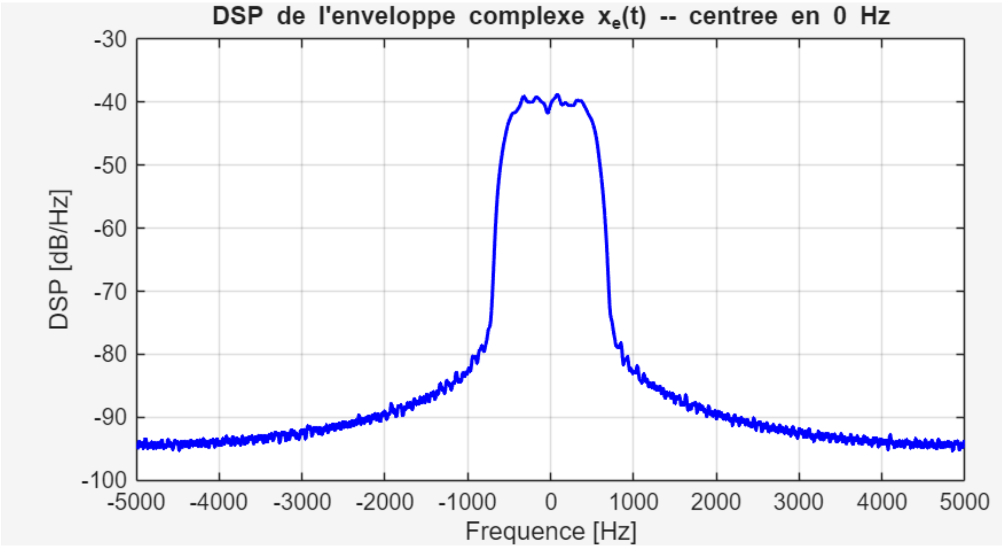

Figure II.2: Power Spectral Density of the complex envelope $x_e(t)$ – Part II, Q2.

Question 3 – Noise-free chain validation

The equivalent lowpass chain is verified without noise. Since no carrier is involved, the matched RRC filter is applied directly to the complex signal $x_e(t)$, and the sampling and decision are performed on the complex samples.

Validation result: BER without noise is **0** in the equivalent lowpass chain, confirming correct implementation.

Question 4 – BER vs E_b/N_0

In the complex domain, the noise is added directly to $x_e(t)$ as a complex Gaussian process. Since the lowpass equivalent works in the complex domain, the noise must have two independent components (one for I and one for Q), each with variance $\sigma^2_{n_I} = \sigma^2_{n_Q}$ given by:

$$\sigma^2_{n_I} = \sigma^2_{n_Q} = \frac{P_{x_e} \cdot N_s}{2\log_2(M) \cdot \frac{E_b}{N_0}} \tag{II.2}$$

The complex noise is therefore: $n(t) = n_I(t) + jn_Q(t)$, where n_I and n_Q are independent.

```
Pxe = mean(abs(xe).^2);
for i = 1:length(EbN0_dB)
    sig  = sqrt((Pxe*Ns) / (2*k*EbN0_lin(i)));
    n_lp = sig * (randn(1,length(xe)) + 1j*randn(1,length(xe)));
    ye   = xe + n_lp;

    re   = filter(h_rrc, 1, ye);
    inst = debut : Ns : (length(re) - n0);
    re_s = re(inst);

    sI_lp = sign(real(re_s));  sQ_lp = sign(imag(re_s));
    b_d   = reshape([(1-sI_lp)/2; (1-sQ_lp)/2], 1, []);
    BER_sim_lp(i) = sum(b_d ~= bits(1:length(b_d))) / length(b_d);
end
```

Listing II.2: Lowpass QPSK BER simulation with complex noise

Question 5 – Constellation at sampler output

The received symbols at the output of the matched filter are plotted in the complex plane for $E_b/N_0 = 5$ dB. The four clusters should be centred around the ideal QPSK constellation points. As E_b/N_0 increases, the clusters contract toward the ideal points.

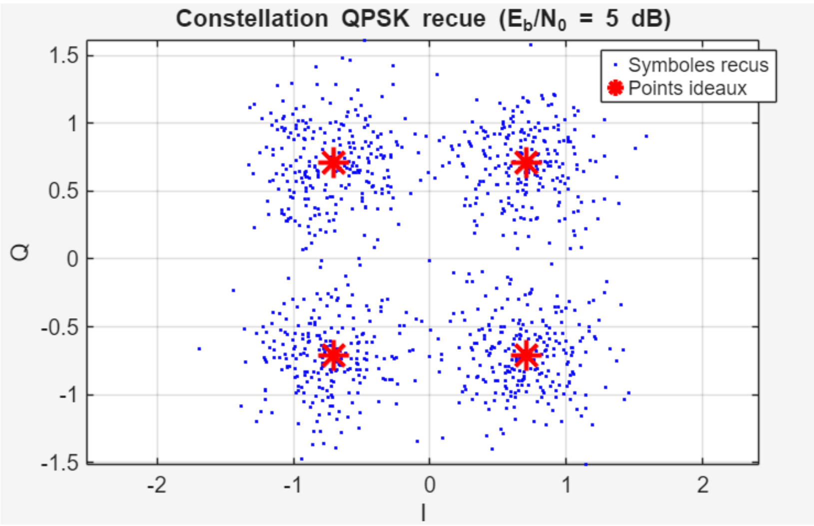

Figure II.3: QPSK constellation at the sampler output for $E_b/N_0 = 5$ dB – Part II, Q5.

Question 6 – Comparison with passband BER

The BER curves of the passband chain (Part I) and the equivalent lowpass chain are plotted together. Both must match the theoretical QPSK BER, confirming the mathematical equivalence between the two representations.

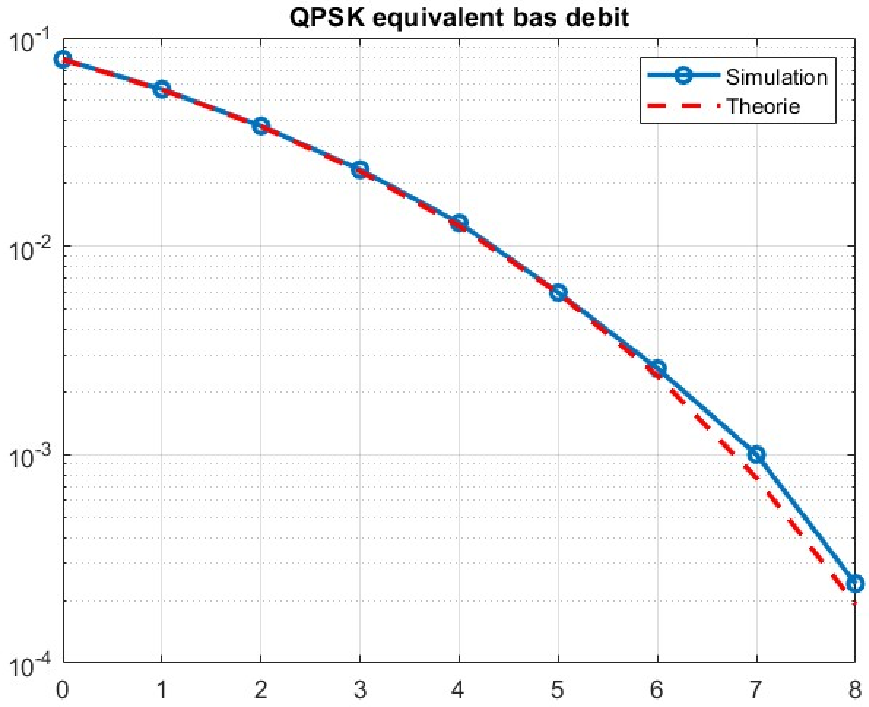

Figure II.4: BER comparison: QPSK passband vs. equivalent lowpass chain – Part II, Q6.

The exact superimposition of the two simulated curves confirms that the lowpass equivalent is a valid and computationally efficient substitute for the passband simulation.

Part III – 4-ASK, 8-PSK and 16-QAM Passband Chains

Overview and parameters

Three additional modulation schemes are implemented with $F_s = 12\,000$ Hz and $R_b = 6\,000$ bps. Table III.1 summarises the key parameters for each modulation.

Table III.1: Modulation parameters for $R_b = 6000$ bps, $\alpha = 0.35$

Modulation	M	k [bit/symb]	R_s [symb/s]	N_s	B [Hz]
QPSK	4	2	1 000	10	675
4-ASK	4	2	3 000	4	2 025
8-PSK	8	3	2 000	6	1 350
16-QAM	16	4	1 500	8	1 013

Constellation design

4-ASK: Four real amplitude levels $\{-3, -1, +1, +3\}$ normalised by $\sqrt{5}$ so that $\mathbb{E}[|s|^2] = 1$. Only the I channel carries information; the Q channel is zero.

8-PSK: Eight uniformly spaced phases on the unit circle. A rotation of $\pi/8$ is applied to improve Gray coding: $s_k = e^{j(2\pi k/8+\pi/8)}$ for $k = 0, \ldots, 7$. Constant envelope makes it suitable for non-linear amplifiers.

16-QAM: A 4×4 square grid with levels $\{-3, -1, +1, +3\}/\sqrt{10}$ on each axis, so that $\mathbb{E}[|s|^2] = 1$. Provides the highest spectral efficiency (4 bit/symbol) at the cost of greater sensitivity to noise.

```
% 4-ASK
const_ask = [-3 -1 1 3] / sqrt(5);

% 8-PSK
const_psk = exp(1j * (2*pi*(0:7)/8 + pi/8));

% 16-QAM
d_qam = [-3 -1 1 3] / sqrt(10);
[dI, dQ] = meshgrid(d_qam, d_qam);
const_qam = dI(:)' + 1j*dQ(:)';
```

Listing III.1: Constellation definitions and pulse shaping for 4-ASK, 8-PSK, 16-QAM

ML Decision

For modulations with more than 2 amplitude levels, a simple sign decision is insufficient. Maximum-Likelihood (ML) detection assigns each received sample to the nearest constellation point in Euclidean distance:

$$\hat{s} = \arg\min_{s_k \in \mathcal{C}} |r - s_k| \tag{III.1}$$

```
[~, id_d] = min(abs(re_s(:) - C_i), [], 2);    % nearest constellation
    point
b_d = reshape(de2bi(id_d-1, k_i, 'left-msb')', 1, []);
```

Listing III.2: ML decision for arbitrary constellation

Question 1 – BER comparison with QPSK

The BER of the three modulations is simulated for E_b/N_0 ranging from 0 to 20 dB. Theoretical references for QPSK and 16-QAM are superimposed:

$$P_e^{\text{QPSK}} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{III.2}$$

$$P_e^{\text{16-QAM}} \approx \frac{3}{4}\,\text{erfc}\left(\sqrt{0.4\,k\,\frac{E_b}{N_0}}\right) \tag{III.3}$$

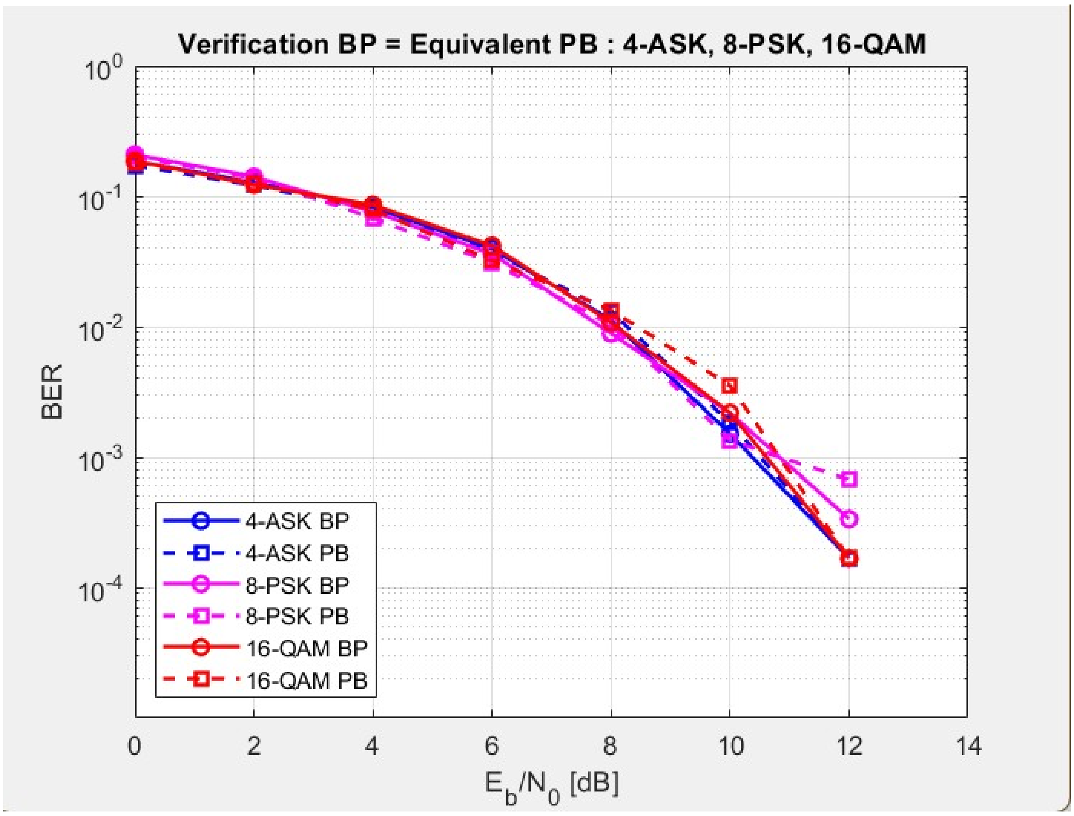

Figure III.1: BER comparison: 4-ASK, 8-PSK and 16-QAM vs. QPSK – Part III, Q1.

Physical interpretation

The performance ordering can be understood as follows. **4-ASK** uses only the I channel, so its decision boundaries are one-dimensional – it is more vulnerable to noise than QPSK for the same spectral efficiency. **8-PSK** places more symbols per unit bandwidth, but its minimum distance between constellation points shrinks as M grows, degrading BER. **16-QAM** achieves the best spectral efficiency (4 bit/symbol) and benefits from the two-dimensional grid, but requires roughly 4 dB more E_b/N_0 than QPSK to reach the same BER floor.

Question 2 – PSD comparison

The PSDs of the three complex envelopes are compared on the same plot. Since $B = R_s(1+\alpha)/2$ and the symbol rates differ, the bandwidths are ordered: 4-ASK $>$ 8-PSK $>$ 16-QAM.

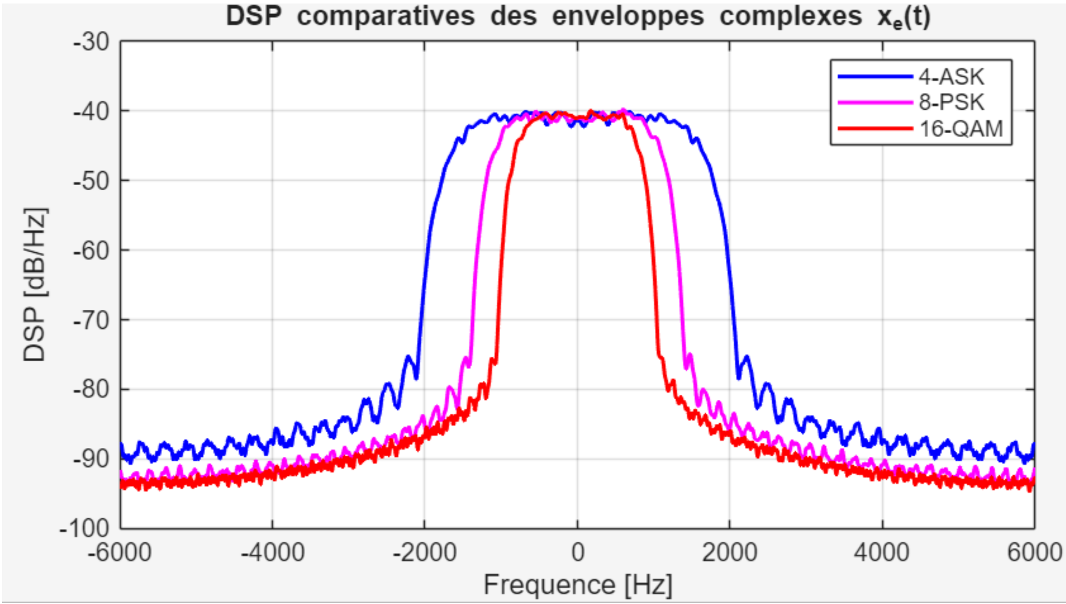

Figure III.2: Comparative PSD of complex envelopes: 4-ASK, 8-PSK and 16-QAM – Part III, Q2.

This illustrates the fundamental trade-off in digital communications: *higher spectral efficiency (more bits per symbol) comes with narrower bandwidth but degraded noise robustness.*

Part IV – Equivalent Lowpass of 4-ASK, 8-PSK and 16-QAM Chains

Overview

The Part III simulations already operate in the equivalent lowpass framework (working directly on x_e). To validate this, a full passband chain is additionally implemented for each modulation: the complex envelope is modulated onto a carrier $f_c = 3000$ Hz, real AWGN noise is added, and the signal is coherently demodulated before the matched filter. The two BER curves – passband and lowpass – must be identical.

Question 1 – Implementation and BER validation

```
fc2   = 3000;    % carrier [Hz]
t_qam = (0:length(xe_qam)-1)/Fs2;
x_bp  = real(xe_qam .* exp(1j*2*pi*fc2*t_qam));    % real passband
    signal

for i = 1:length(EbN0_dB3)
    Px_bp = mean(abs(x_bp).^2);
    sig   = sqrt((Px_bp*Ns_qam) / (2*k_qam*EbN0_lin3(i)));
    y_bp  = x_bp + sig*randn(1, length(x_bp));    % real noise

    yI_bp = y_bp .* (2*cos(2*pi*fc2*t_qam));
    yQ_bp = y_bp .* (-2*sin(2*pi*fc2*t_qam));
    rI_bp = filter(h_qam, 1, yI_bp);
    rQ_bp = filter(h_qam, 1, yQ_bp);

    inst  = debut_qam : Ns_qam : (length(rI_bp) - n0_qam);
    re_bp = rI_bp(inst) + 1j*rQ_bp(inst);    % reconstructed complex
        samples

    [~, id_d] = min(abs(re_bp(:) - const_qam), [], 2);
    b_d = reshape(de2bi(id_d-1, k_qam, 'left-msb')', 1, []);
    BER_bp16(i) = sum(b_d ~= bits_qam(1:length(b_d))) / length(b_d);
end
```

Listing IV.1: Passband chain for 16-QAM (used as example for all three modulations)

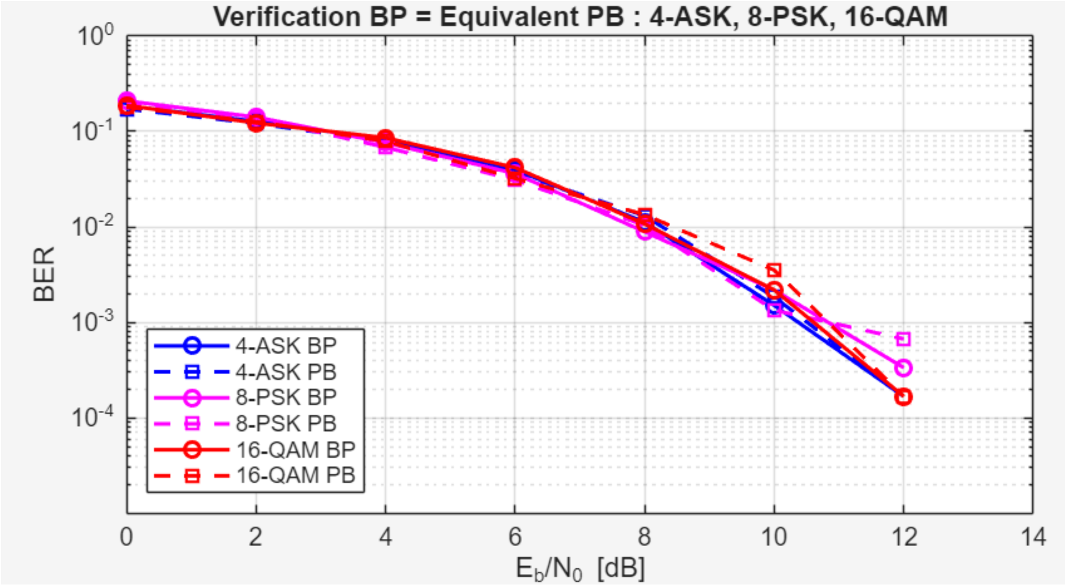

Figure IV.1: Verification of BP = equivalent lowpass: 4-ASK, 8-PSK and 16-QAM – Part IV, Q1.

The console output reports the maximum absolute difference between the two BER vectors for each modulation. This gap should be close to zero (statistical fluctuation only).

Question 2 – Synthesis and comparison

Table IV.1 provides a comprehensive comparison of all modulation schemes studied in this project, including spectral efficiency, occupied bandwidth, and BER performance at 8 dB.

Table IV.1: Comparison of modulation schemes ($R_b = 6000$ bps, $\alpha = 0.35$)

Modulation	k [bit/symb]	R_s [symb/s]	B [Hz]	Spectral Efficiency	BER at 8 dB
QPSK	2	1 000	675	2 bit/s/Hz	$\approx 2 \times 10^{-4}$
4-ASK	2	3 000	2 025	2 bit/s/Hz	worse than QPSK
8-PSK	3	2 000	1 350	3 bit/s/Hz	worse than QPSK
16-QAM	4	1 500	1 013	4 bit/s/Hz	$\approx 5 \times 10^{-3}$

Discussion

QPSK offers the best BER performance for a fixed E_b/N_0 and remains the reference modulation. 4-ASK achieves the same spectral efficiency as QPSK but degrades BER significantly, as its symbols lie only on the real axis and the minimum Euclidean distance is reduced. 8-PSK offers higher spectral efficiency (3 bit/symbol) while maintaining constant envelope, making it attractive for non-linear channels such as satellite links, but at the cost of higher E_b/N_0 requirements. 16-QAM provides the highest spectral efficiency (4 bit/symbol) and the narrowest occupied bandwidth, but requires approximately 4 dB more power than QPSK to achieve an equivalent BER.

The results confirm the fundamental principle of digital communications: increasing spectral efficiency always comes at the cost of reduced noise immunity. The choice of modulation scheme must therefore balance the available power margin against the required data rate and bandwidth.

Conclusion

This project provided a complete implementation and analysis of digital passband transmission systems in MATLAB. The following key results were established:

- The QPSK passband chain was implemented and validated: the BER without noise is exactly zero, and the simulated BER closely matches the theoretical expression $\frac{1}{2}\text{erfc}(\sqrt{E_b/N_0})$.
- The equivalent lowpass formulation was verified to produce identical BER performance, confirming its validity as a computationally efficient substitute regardless of carrier frequency.
- Among the modulations compared at $R_b = 6000$ bps, QPSK provides the best BER-to-power trade-off. 16-QAM achieves the highest spectral efficiency (4 bit/symbol) and the narrowest occupied bandwidth, but at the cost of a $\sim$4 dB power penalty.
- The passband-to-lowpass equivalence was verified for all three additional modulations (4-ASK, 8-PSK, 16-QAM), with near-perfect superimposition of the corresponding BER curves.